INTERMEDIATE ALGEBRA

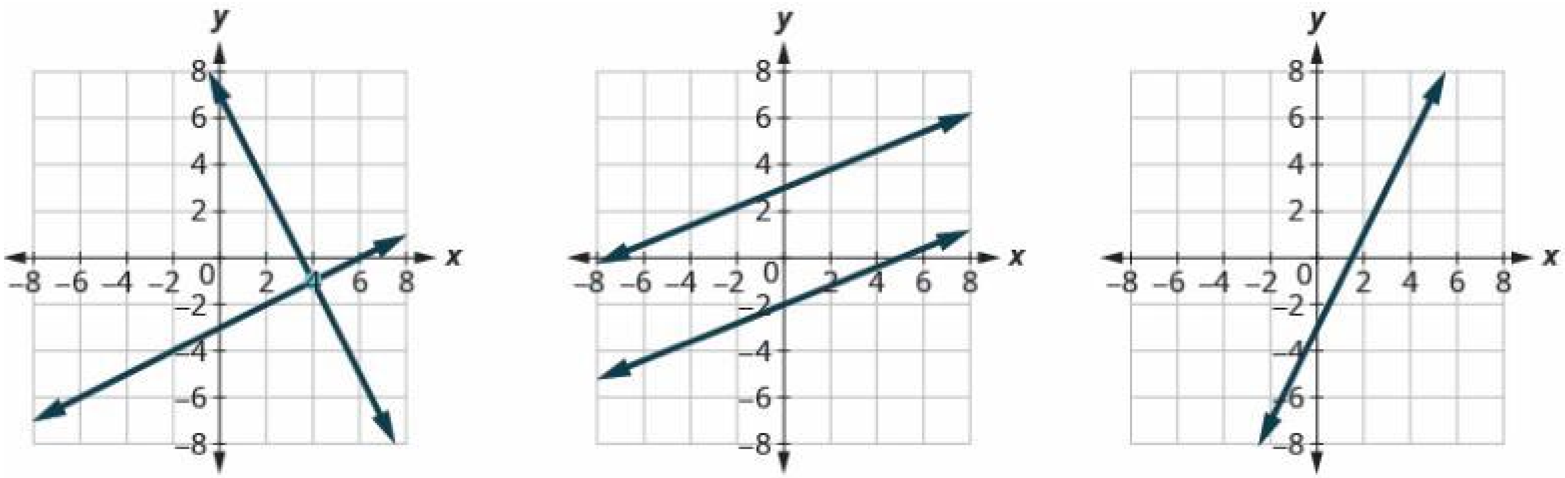
Chapter 4 SYSTEMS OF LINEAR EQUATIONS

PowerPoint Image Slideshow

FIGURE 4.1

In the future, car drivers may become passengers because cars will be able to drive themselves. (credit: jingoba/Pixabay)

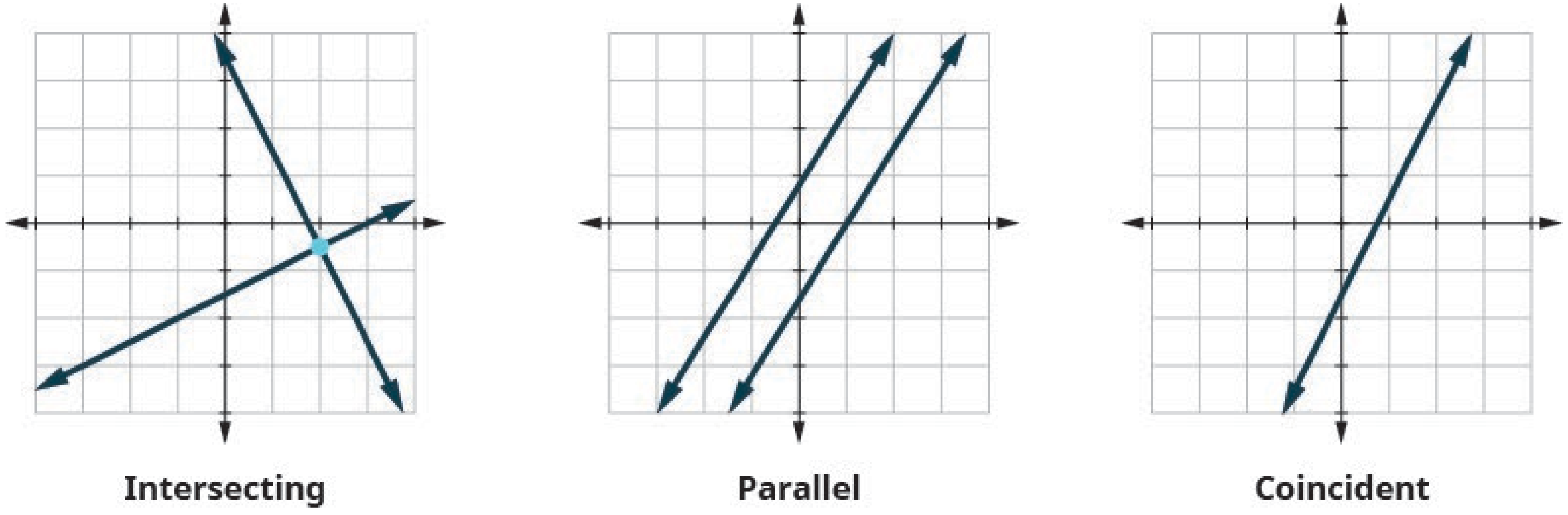
FIGURE 4.2



The lines intersect.
Intersecting lines have one point in common.
There is one solution to this system.

The lines are parallel.
Parallel lines have no points in common.
There is no solution to this system.

Both equations give the same line.
Because we have just one line, there are infinitely many solutions.

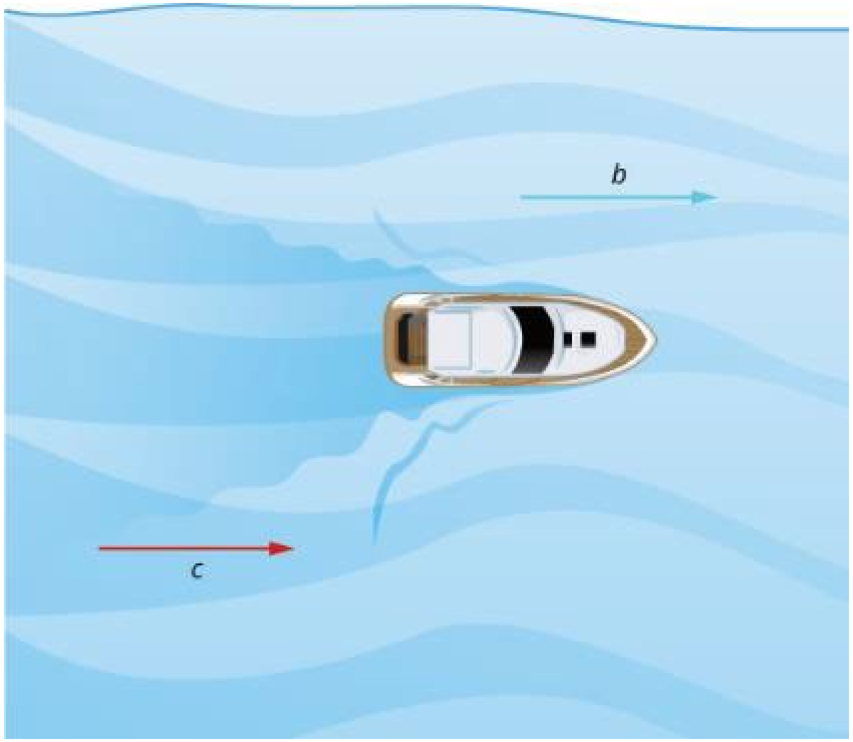

Intersecting
Parallel
Coincident

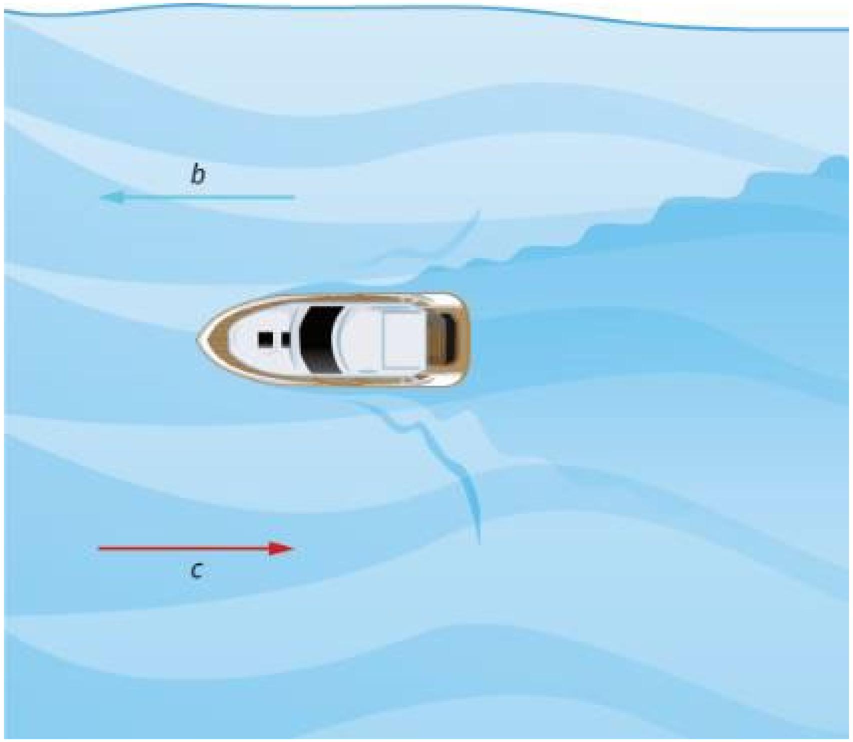
$$\begin{cases} -2(x + 4y) = -2(2) \\ 2x + 5y = -2 \end{cases}$$

$$\begin{cases} -2x - 8y = -4 \\ 2x + 5y = -2 \end{cases}$$

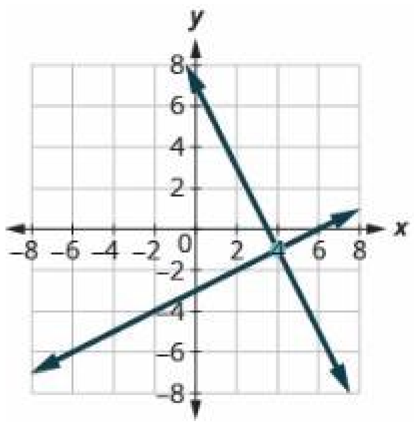
$$\begin{cases} -2x - 8y = -4 \\ 2x + 5y = -2 \end{cases}$$

$$-3y = -6$$


b
c


b
c

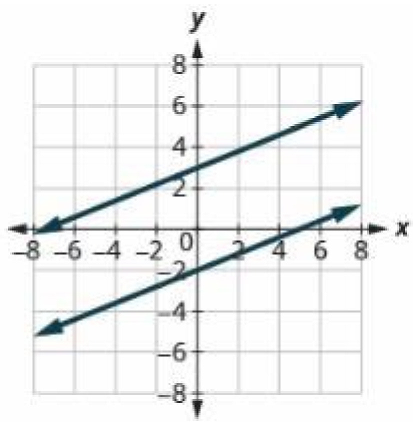
	Number	• Value	= Total Value
nickels			
dimes			



One solution

The lines intersect.
Intersecting lines have one point in common.
There is one solution to this system.

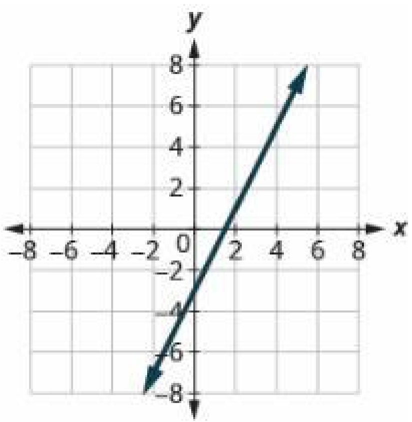
Consistent
Independent



No solution

The lines are parallel.
Parallel lines have no points in common.
There is no solution to this system.

Inconsistent



Infinitely many solutions

Both equations give the same line.
Because we have just one line, there are infinitely many solutions.

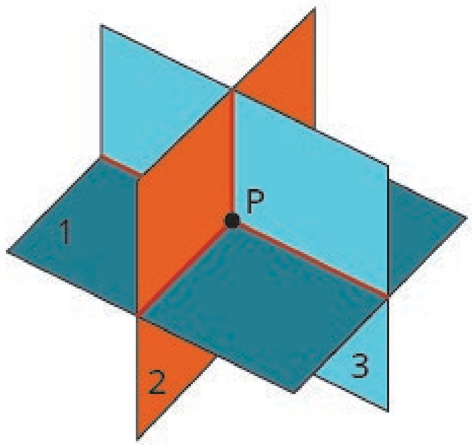
Consistent
Dependent

One solution

Consistent system and Independent equations

The 3 planes intersect.

The three intersecting planes have one point in common.

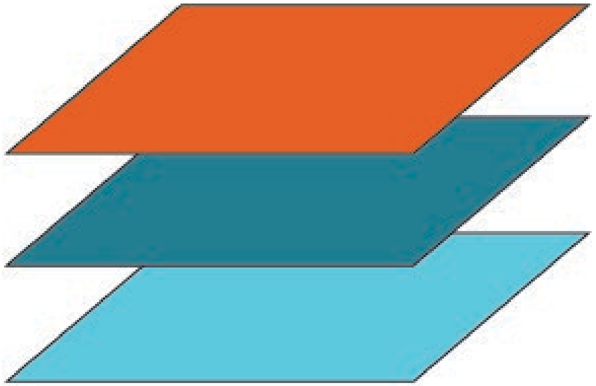


No solution

Inconsistent system

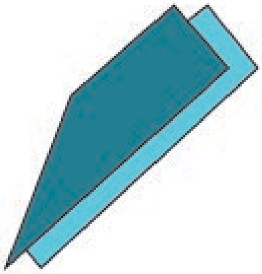
The planes are parallel.

Parallel planes have no points in common.



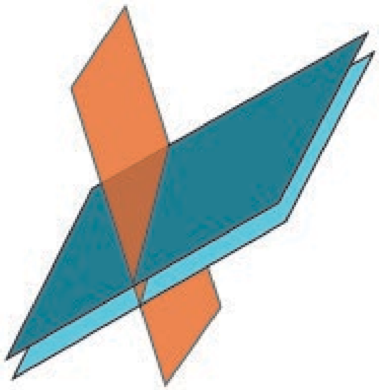
Two planes are coincident and parallel to the third plane.

The planes have no points in common.



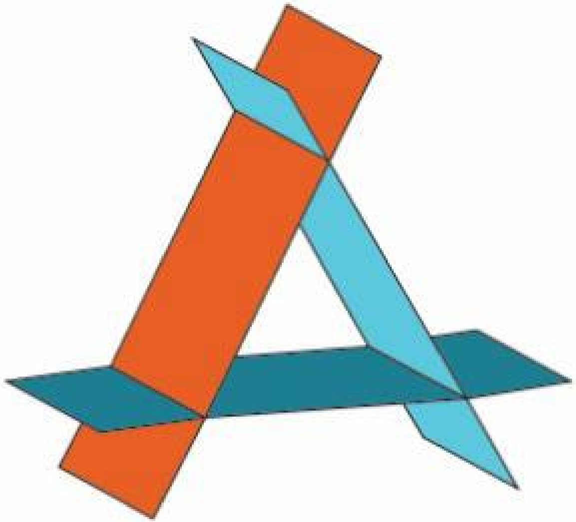
Two planes are parallel and each intersect the third plane.

The planes have no points in common.



Each plane intersects the other two, but all three share no points.

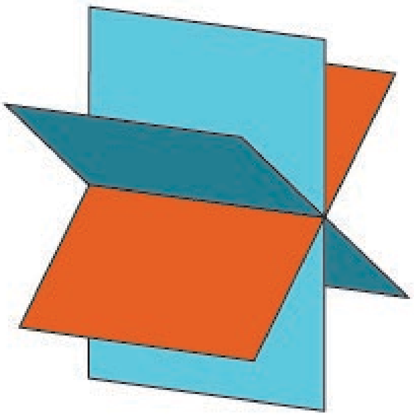
The planes have no points in common.



Infinitely many solutions
Consistent system and dependent equations

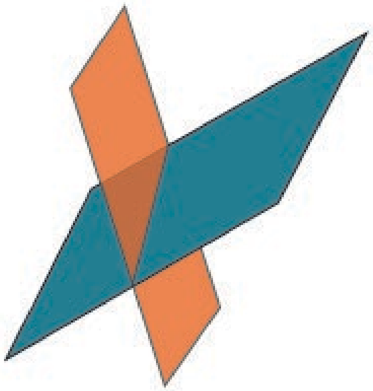
Three planes intersect in one line.

There is just one line, so there are infinitely many solutions.



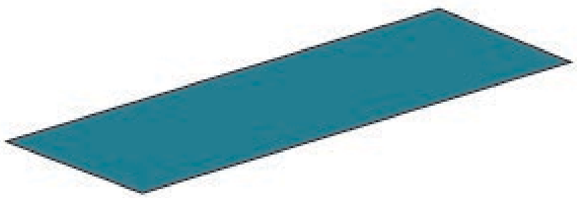
Two planes are coincident and intersect the third plane in a line.

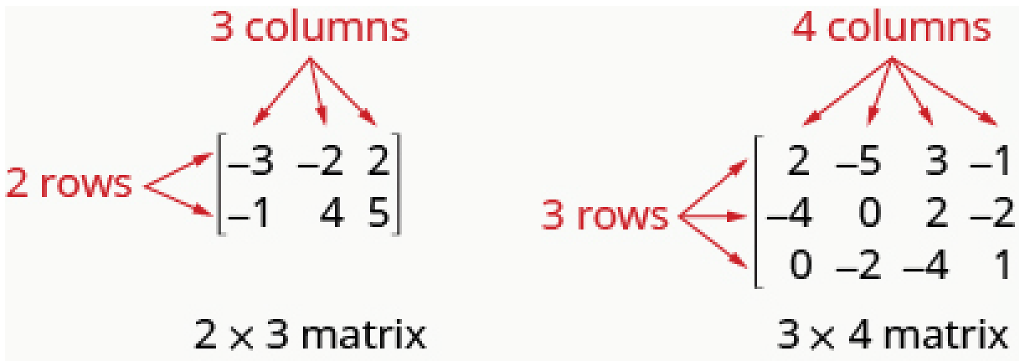
There is just one line, so there are infinitely many solutions.



Three planes are coincident.

There is just one plane, so there are infinitely many solutions.




3 columns
2 rows
−3 −2 2
−1 4 5
2 × 3 matrix
4 columns
3 rows
2 −5 3 −1
−4 0 2 −2
0 −2 −4 1
3 × 4 matrix

$$\begin{cases} 3x - y = -3 \\ 2x + 3y = 6 \end{cases} \longrightarrow \left[\begin{array}{cc|c} 3 & 1 & -3 \\ 2 & 3 & 6 \end{array}\right]$$

coefficients of x

coefficients of y

constants

$$\left[\begin{array}{cc|c} 5 & -3 & -1 \\ 2 & -1 & 2 \end{array}\right] \longrightarrow \begin{array}{c} R_2 \\ R_1 \end{array} \left[\begin{array}{cc|c} 2 & -1 & 2 \\ 5 & -3 & -1 \end{array}\right]$$

$$\left[\begin{array}{cc|c} 5 & -3 & -1 \\ 2 & -1 & 2 \end{array}\right] \longrightarrow -3R_2 \left[\begin{array}{cc|c} 5 & -3 & -1 \\ -6 & 3 & -6 \end{array}\right]$$

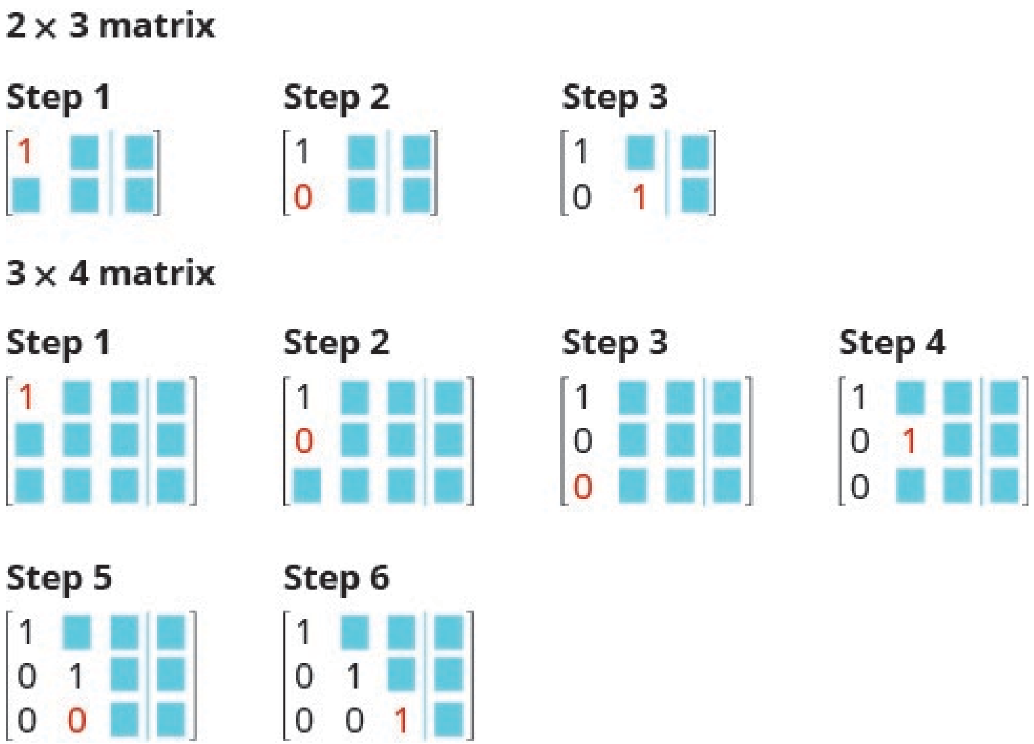
$$\left[\begin{array}{cc|c} 5 & -3 & -1 \\ 2 & -1 & 2 \end{array}\right] \xrightarrow{-3R_2 + R_1} \left[\begin{array}{cc|c} -1 & 0 & -7 \\ 2 & -1 & 2 \end{array}\right]$$

$$\begin{cases} x - y = 2 \\ 4x - 8y = 0 \end{cases} \xrightarrow{\text{multiply the first equation by } -4} \begin{cases} -4x + 4y = -8 \\ 4x - 8y = 0 \end{cases} \xrightarrow{\text{then add}} \begin{cases} -4x + 4y = -8 \\ 4x - 8y = 0 \end{cases}$$

$$-4y = -8$$

$$\left[\begin{array}{cc|c} 1 & a & b \\ 0 & 1 & c \end{array}\right] \qquad \left[\begin{array}{ccc|c} 1 & a & b & d \\ 0 & 1 & c & e \\ 0 & 0 & 1 & f \end{array}\right]$$

a, b, c, d, e, f are real numbers


2 × 3 matrix
Step 1
Step 2
Step 3
3 × 4 matrix
Step 1
Step 2
Step 3
Step 4
Step 5
Step 6

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} = ad - bc$$

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \text{ minor of } a_1 \quad \begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix}$$

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \quad \text{minor of } b_2 \quad \begin{vmatrix} a_1 & c_1 \\ a_3 & c_3 \end{vmatrix}$$

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = a_1 \underbrace{\begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix}}_{\text{minor of } a_1} - b_1 \underbrace{\begin{vmatrix} a_2 & c_2 \\ a_3 & c_3 \end{vmatrix}}_{\text{minor of } b_1} + c_1 \underbrace{\begin{vmatrix} a_2 & b_2 \\ a_3 & b_3 \end{vmatrix}}_{\text{minor of } c_1}$$

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = a_1 \underbrace{\begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix}}_{\text{minor of } a_1} - b_1 \underbrace{\begin{vmatrix} a_2 & c_2 \\ a_3 & c_3 \end{vmatrix}}_{\text{minor of } b_1} + c_1 \underbrace{\begin{vmatrix} a_2 & b_2 \\ a_3 & b_3 \end{vmatrix}}_{\text{minor of } c_1}$$

$$\begin{vmatrix} + & - & + \\ - & + & - \\ + & - & + \end{vmatrix}$$

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = a_1 \underbrace{\begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix}}_{\text{minor of } a_1} - b_1 \underbrace{\begin{vmatrix} a_2 & c_2 \\ a_3 & c_3 \end{vmatrix}}_{\text{minor of } b_1} + c_1 \underbrace{\begin{vmatrix} a_2 & b_2 \\ a_3 & b_3 \end{vmatrix}}_{\text{minor of } c_1}$$

$$x = \frac{D_x}{D} \text{ and } y = \frac{D_y}{D}$$

where $D = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$ use the coefficients of the variables.

$D_x = \begin{vmatrix} k_1 & b_1 \\ k_2 & b_2 \end{vmatrix}$ replace the x coefficients with the constants.

$D_y = \begin{vmatrix} a_1 & k_1 \\ a_2 & k_2 \end{vmatrix}$ replace the y coefficients with the constants.

$$\begin{cases} a_1 x + b_1 y = k_1 \\ a_2 x + b_2 y = k_2 \end{cases}$$

Coefficients

$$D = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$$

Coefficient of x

Coefficient of y

$$\begin{cases} a_1x + b_1y = k_1 \\ a_2x + b_2y = k_2 \end{cases}$$

$$D_x = \begin{vmatrix} k_1 & b_1 \\ k_2 & b_2 \end{vmatrix}$$

Constants

Coefficients of y

$$\begin{cases} a_1x + b_1y = k_1 \\ a_2x + b_2y = k_2 \end{cases}$$

$$D_y = \begin{vmatrix} a_1 & k_1 \\ a_2 & k_2 \end{vmatrix}$$

Coefficients of x

Constants

$$x = \frac{D_x}{D},\ y = \frac{D_y}{D} \text{ and } z = \frac{D_z}{D}$$

where $D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ use the coefficients of the variables.

$D_x = \begin{vmatrix} k_1 & b_1 & c_1 \\ k_2 & b_2 & c_2 \\ k_3 & b_3 & c_3 \end{vmatrix}$ replace the x coefficients with the constants.

$D_y = \begin{vmatrix} a_1 & k_1 & c_1 \\ a_2 & k_2 & c_2 \\ a_3 & k_3 & c_3 \end{vmatrix}$ replace the y coefficients with the constants.

$D_z = \begin{vmatrix} a_1 & b_1 & k_1 \\ a_2 & b_2 & k_2 \\ a_3 & b_3 & k_3 \end{vmatrix}$ replace the z coefficients with the constants.